

# SPECTRUM OF THE

## TRANSMITTED SIGNAL

$$x(t) = \sum_{k=-\infty}^{\infty} a_k P_T(t - kT)$$

$k^{\text{th}}$  symbol

pulse shifted by  $kT$

Can be any pulse  $p(t)$ .

$k^{\text{th}}$  bit = 0 or 1

random

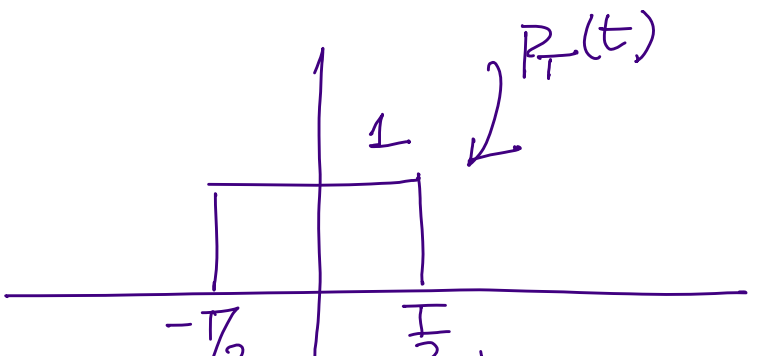
$a_k = A$  or  $-A$

randomly

Aim: To characterize spectrum of transmitted signal  $x(t)$

$$P_T(t) = \begin{cases} 1 & |t| \leq T/2 \\ 0 & \text{otherwise} \end{cases}$$

$P_T(t)$   
 $\updownarrow$   
 $T \text{sinc} fT$   
 $= \frac{T \sin(\pi fT)}{\pi fT}$



Since  $a_k$  is random,

$$P(a_k) = \begin{matrix} \nearrow A = \frac{1}{2} \\ \searrow -A = -\frac{1}{2} \end{matrix}$$

Equiprobable.

$$\begin{aligned}
 E\{a_k\} &= A \times P_r(a_k = A) \\
 &\quad + (-A) \times P_r(a_k = -A) \\
 &= A \times \frac{1}{2} + (-A) \times \frac{1}{2}
 \end{aligned}$$

$$\boxed{E\{a_k\} = 0}$$

Also assume symbols  $a_k$  are

IID — Independent  
Identically Distributed.

$$E\{a_k a_m\} = \begin{matrix} E\{a_k\} \\ 0 \end{matrix} \times \begin{matrix} E\{a_m\} \\ 0 \end{matrix} \quad \text{if } k \neq m$$

$$= 0$$

uncorrelated random variables.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k p_T(t - kT)$$

Average value of this?

$$E\{x(t)\} = E\left\{\sum_{k=-\infty}^{\infty} a_k p_T(t - kT)\right\}$$

$$= \sum_{k=-\infty}^{\infty} \underbrace{E\{a_k\}}_0 p_T(t - kT)$$

$$E\{x(t)\} = 0$$

Average value of Transmitted signal  $x(t)$  is 0.

F.T.  $\{x(t)\}$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

$$\begin{aligned} E\{X(f)\} &= E\left\{ \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \right\} \\ &= \int_{-\infty}^{\infty} \underbrace{E\{x(t)\}}_0 e^{-j2\pi ft} dt \end{aligned}$$

$$\boxed{E\{X(f)\} = 0} !$$

Average value of spectrum is 0.

Does NOT mean spectrum is 0.

$$\underbrace{x(t)}_{\text{random}} = \sum_{k=-\infty}^{\infty} \underbrace{a_k}_{\text{random}} p_T(t - kT)$$

How to measure the spectral content of a random signal?

Power Spectral Density (PSD)

Spectral Distribution of power of a random signal.

We have to first compute the auto-correlation

$$R_{xx}(\tau) = E \{ x(t) x(t+\tau) \}$$

For a WSS (Wide Sense Stationary) Process,

Auto-correlation  $\xleftrightarrow{FT}$  PSD.